

MARK SCHEME for the May/June 2014 series

4037 ADDITIONAL MATHEMATICS

4037/22

Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Page 2	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – May/June 2014	4037	22

<p>1</p>	<p>rationalise the denominator to get $\frac{(2+\sqrt{5})^2(\sqrt{5}+1)}{5-1}$ or better</p> <p>squaring to get $\frac{(4+4\sqrt{5}+5)(\sqrt{5}+1)}{\text{their4}}$ or better</p> <p>$\frac{29}{4} + \frac{13}{4}\sqrt{5}$ oe isw</p>	<p>M1</p> <p>M1</p> <p>A1 + A1</p>	<p>or squaring to get $\frac{(4+4\sqrt{5}+5)}{\sqrt{5}-1}$ or better</p> <p>or rationalising the denominator to get $\frac{\text{their}(9+4\sqrt{5})(\sqrt{5}+1)}{5-1}$ or better</p> <p>correct simplification</p> <p>Allow $\frac{29+13\sqrt{5}}{4}$ etc.</p>
<p>2</p>	<p>Correctly eliminate y</p> <p>$2x^2 + (k-9)x + 2 [= 0]$ oe</p> <p>Use $b^2 - 4ac$ oe</p> <p>Reach $\text{their}(k-9 = \pm 4)$ or solves $\text{their}(k^2 - 18k + 65) = 0$</p> <p>$k = 5$ and 13 cao</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>$-kx + 2 = 2x^2 - 9x + 4$ oe</p> <p>allow even if x terms not collected; condone $\dots = y$ provided later work implies it should be 0</p> <p>must be applied to a 3 term quadratic expression containing k as a coefficient; condone < 0 etc.</p> <p>condone $9 - k = \pm 4$; condone an inequality at this stage</p> <p>mark final answer, do not isw; A0 if inequalities for final answers</p>

Page 3	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – May/June 2014	4037	22

3	(i)	$3(-1)^3 - 14(-1)^2 - 7(-1) + d = 0$ with completion to $d = 10$	B1	at least $-3 - 14 + 7 + d = 0$, $d = 10$; N.B. = 0 must be seen or implied by $\dots = d$ or $\dots = -d$, may be seen in following step. or convincingly showing $3(-1)^3 - 14(-1)^2 - 7(-1) + 10 = 0$; at least $-3 - 14 + 7 + 10 = 0$ or correct synthetic division at least as far as
	(ii)	$3x^2 - 17x + 10$ isw or $a = 3, b = -17, c = 10$ isw	B2, 1, 0	-1 each error; must be seen or referenced in (ii) even if found in (i) or (iii)
	(iii)	$(x+1)(x-5)(3x-2)$ $-1, 5, \frac{2}{3}$	M1 A1	for factorising quadratic ft correct; condone omission of $(x+1)$ or for ft correct use of formula or ft correct completing the square If M0 then SC1 for all three roots stated without working or verified/found by trials

Page 4	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – May/June 2014	4037	22

<p>4 (i)</p> <p>(ii)</p>	<p>$12\left(x - \frac{1}{4}\right)^2 + \frac{17}{4}$ isw</p> <p>their $\frac{4}{17}$ or their 0.235</p> <p>their $x = \frac{1}{4}$ oe</p>	<p>B3, 2, 1, 0</p> <p>B1ft</p> <p>B1ft</p>	<p>one mark for each of p, q, r correct in a correctly formatted expression; allow correct equivalent values;</p> <p>If B0 then SC2 for $12\left(x - \frac{1}{4}\right) + \frac{17}{4}$</p> <p>or</p> <p>SC1 for correct 3 values seen in incorrect format e.g.</p> <p>$12\left(x - \frac{1}{4}x\right) + \frac{17}{4}$ or</p> <p>$12\left(x^2 - \frac{1}{4}\right) + \frac{17}{4}$</p> <p>or for a correct completed square form of the original expression in a different but correct format. e.g.</p> <p>$3\left(2x - \frac{1}{2}\right)^2 + \frac{17}{4}$</p> <p>strict ft ; their $\frac{4}{17}$ must be a proper fraction or decimal rounded to 3sig figs or more or truncated to 4 figs or more</p> <p>strict ft ; x must be correctly attributed</p>
<p>5 (i)</p> <p>(ii)</p>	<p>$1 - 20x + 160x^2$</p> <p>$a + (\text{their} - 20) = -23$ soi</p> <p>$a = -3$</p> <p>$b + (\text{their} - 20)a + (\text{their}160) = 222$ soi</p> <p>$b = 2$</p>	<p>B2, 1, 0</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>-1 each error</p> <p>if B0 then M1 for 3 correct terms seen; may be unsimplified e.g.</p> <p>$1, 5(-4x), \frac{5 \times 4}{2}(-4x)^2$</p> <p>condone sign errors only; must be their -20 from (i)</p> <p>validly obtained</p> <p>condone sign errors only ; must be their -20 and their160 from (i) and their a if used</p> <p>validly obtained</p>

Page 5	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – May/June 2014	4037	22

<p>6 (a) (i) 1</p> <p>(ii) $x = -1$ or -2</p> <p>(b) $\frac{\log_3 5}{\log_3 a}$ seen or implied</p> <p>$2 \log_3 15 = \log_3 15^2$ seen or implied</p> <p>$\log_3 15^2 - \log_3 5 = \log_3 \left(\frac{15^2}{5} \right)$</p> <p>$\log_3 45$ cao</p>		<p>B1</p> <p>B1 + B1</p> <p>B1*</p> <p>B1</p> <p>B1dep*</p> <p>B1</p>	<p>as final answers</p> <p>may be implied by $2 \log_3 15 - \log_3 5$</p> <p>not from wrong working</p> <p>must be 45 not e.g. $\frac{225}{5}$; with no wrong working seen</p>
<p>7 (i) $x^4(3e^{3x}) + 4x^3e^{3x}$ isw</p> <p>(ii) $\frac{1}{2 + \cos x} \times (-\sin x)$ isw</p> <p>(iii) $\frac{d}{dx}(\sin x) = \cos x$ soi</p> <p>$\frac{d}{dx}(1 + \sqrt{x}) = \frac{1}{2}x^{-\frac{1}{2}}$ soi</p> <p>$\frac{(1 + \sqrt{x}) \text{ their } \cos x - \left(\text{their } \frac{1}{2} x^{-\frac{1}{2}} \right) \sin x}{(1 + \sqrt{x})^2}$ isw</p>		<p>B1 + B1</p> <p>B2</p> <p>B1</p> <p>B1</p> <p>B1ft</p>	<p>each term of the sum correct; must be a sum of two terms</p> <p>or B1 for $\frac{1}{2 + \cos x} \times (k \pm \sin x)$ and k a constant</p> <p>for correct form of quotient rule ft their $\cos x$ and their $\frac{1}{2}x^{-\frac{1}{2}}$;</p> <p>allow correct use of product and chain rules to obtain</p> <p>$\sin x \left(- (1 + \sqrt{x})^{-2} \times \frac{1}{2} x^{\frac{1}{2}} \right) +$ $\cos x (1 + \sqrt{x})^{-1}$ oe</p>

Page 6	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – May/June 2014	4037	22

8	<p>Substitution of either $x - 5$ or $y + 5$ into equation of curve and brackets expanded</p> <p>$2x^2 - 8x - 10 [= 0]$ or $2y^2 + 12y [= 0]$ obtained</p> <p>Solving their quadratic</p> <p>$(-1, -6)$ oe and $(5, 0)$ oe isw</p> <p>$\sqrt{72}$ or $6\sqrt{2}$ cao isw</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1*+A1*</p> <p>B1dep*</p>	<p>condone one sign error in either equation of curve or expansion of brackets; condone omission of $= 0$, BUT $x - 5$ or $y + 5$ must be correct</p> <p>dep on a valid substitution attempt</p> <p>or A1 for correct pair of x coordinates or correct pair of y coordinates</p>
9	<p>(i)</p> <p>$[y =] \frac{(2x+1)^{\frac{3}{2}}}{2 \times \frac{3}{2}} (+c)$ oe</p> <p>$10 = \frac{2}{6} (2(4)+1)^{\frac{3}{2}} + c$ oe</p> <p>$y = \frac{(2x+1)^{\frac{3}{2}}}{2 \times \frac{3}{2}} + c$ seen and $c = 1$ or</p> <p>$y = \frac{(2x+1)^{\frac{3}{2}}}{2 \times \frac{3}{2}} + 1$ isw</p> <p>(ii)</p> <p>$\int \left(\frac{1}{3} (2x+1)^{\frac{3}{2}} + 1 \right) dx = \frac{1}{15} (2x+1)^{\frac{5}{2}} + x (+const)$</p> <p>$\left[\frac{1}{15} (2x+1)^{\frac{5}{2}} + x \right]_0^{1.5} =$</p> <p>$\left[\frac{1}{15} (2(1.5)+1)^{\frac{5}{2}} + (1.5) \right] - \left[\frac{1}{15} (2(0)+1)^{\frac{5}{2}} + 0 \right]$</p> <p>$\frac{107}{30}$ oe isw</p>	<p>B2</p> <p>M1</p> <p>A1</p> <p>B1 + B1</p> <p>B1ft</p> <p>M1</p> <p>A1</p>	<p>or B1 for $(2x+1)^{\frac{1}{2}+1}$</p> <p>for valid attempt to find c; condone slips e.g. omission of power or sign error</p> <p>must have $y = \dots$; condone $f(x) = \dots$</p> <p>B1 for $(2x+1)^{\frac{3}{2}+1}$,</p> <p>B1 for $\frac{1}{15} (2x+1)^{\frac{5}{2}}$</p> <p>B1 ft their c from (i) provided $c \neq 0$</p> <p>for a genuine attempt to find $F(1.5) - F(0)$ in an attempt to integrate <i>their y</i>; if their $F(0)$ is 0 must see at least their $F(1.5) - 0$; condone $+c$ as long as their c is not numerical.</p> <p>if decimal 3.57 or more accurate e.g. 3.566</p>

Page 7	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – May/June 2014	4037	22

10	(i)	Taking logs of both sides $\log y = \log A + x \log b$	M1	any base; must be an explicitly correct statement
	(ii)	b : awrt 3 to one sf isw or awrt 4 to one sf isw A : awrt 0.5 to one sf	A1	correct form; any base; no recovery from incorrect method steps
	(iii)	Evidence of graph used at $\ln y = 5.4$ soi awrt 4.4 to two sf	B2 B2 M1 A1	or M1 for $b = e^{\text{their gradient}}$ soi; their gradient must be correctly evaluated as rise/run or B1 for $A = e^{-0.6}$ or SC1 for $A = e^{-0.3} = 0.7$ (giving an awrt 0.7) or $\frac{220}{\text{their}0.5} = (\text{their}4)^x$ or $5.39\dots = \text{their}(1.4)x + \text{their} -0.6$ or $\ln(220) = x \ln(\text{their}4) + \ln(\text{their}0.5)$

Page 8	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – May/June 2014	4037	22

11	(i)	$f(x) > 3$ or $[f(x) \in](3, \infty)$	B1	condone $y > 3$
	(ii)	$x + 1 = 2^y$ $f^{-1}(x) = \log_2(x + 1)$	M1 A1	or $y + 1 = 2^x$ mark final answer or $\log_2(y + 1) = x$ and $f^{-1}(x) = \log_2(x + 1)$ or for $f^{-1}(x) = \frac{\log(x + 1)}{\log 2}$ (any base for this form)
	(iii)	Domain $x > 3$ Range $f^{-1}(x) > 2$ $2^x(2^x - 1)$ oe isw $2^x(2^x - 1) = 0$ leading to $2^x = 0$, impossible oe $2^x = 1 \Rightarrow x = 0$ 0 is not in the domain (and so $gf(x) = 0$ has no solutions)	B1ft B1 B1 B1 M1 A1	ft their range of f provided mathematically valid inequality or interval condone $f(x) > 2$ or $y > 2$ e.g. $(2^x - 1)^2 + (2x - 1)$ or $2^{2x} - 2 \times 2^x + 1 + 2^x - 1$ or $2^x = 0$ which is outside domain of gf or $2^x(2^x - 1) = 2^{2x} - 2^x = 0$ $[2^{2x} = 2^x] \Rightarrow x = 0$

Page 9	Mark Scheme	Syllabus	Paper
	GCE O LEVEL – May/June 2014	4037	22

<p>12 (i)</p> <p>$\frac{dy}{dx} = 3x^2 - 18x + 24$</p> <p>Solving their $3x^2 - 18x + 24 \geq 0$ by factorising or quadratic formula or completing the square</p> <p>Critical values 2 and 4 $x \leq 2, x \geq 4$</p> <p>(ii)</p> <p>Evaluating their $\frac{dy}{dx}$ at $x = 3$</p> <p>Use of $m_1 m_2 = -1$ to get $m_{normal} = -\frac{1}{their(-3)}$</p> <p>$y = 18$ soi</p> <p>$y - their18 = \left(their \frac{1}{3} \right) (x - 3)$ or</p> <p>$y = their \frac{1}{3} x + c$ and $c = their17$ isw</p> <p>$P(0, 17)$ cao</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>B1</p> <p>A1ft</p> <p>B1</p>	<p>attempt at differentiation resulting in quadratic expression with two terms correct; allow = or \leq or $<$ or $>$ or ≥ 0 omitted here.</p> <p>A0 if spurious attempt to combine; mark final answer</p> <p>must be explicit statement of gradient of normal ; may be seen in equation</p> <p>ft their m provided a genuine attempt at m_{normal} ; no ft if $m = their m_{tangent}$</p>
--	--	---